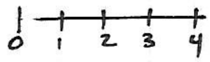
AP Calculus AB

Approx. Area Using Riemann Sums #2

1) $f(x) = (x-1)^2$ on $[0,4]$



$\Delta x = \frac{4}{4} = 1$

a) $L_4 = f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$
$= (0-1)^2 + (1-1)^2 + (2-1)^2 + (3-1)^2$

b) $R_4 = f(4)\cdot 1 + f(3)\cdot 1 + f(2)\cdot 1 + f(1)\cdot 1$
$= (4-1)^2 + (3-1)^2 + (2-1)^2 + (1-1)^2$

c) $M_4 = f(\frac{1}{2})\cdot 1 + f(\frac{3}{2})\cdot 1 + f(\frac{5}{2})\cdot 1 + f(\frac{7}{2})\cdot 1$
$= (\frac{1}{2}-1)^2 + (\frac{3}{2}-1)^2 + (\frac{5}{2}-1)^2 + (\frac{7}{2}-1)^2$

d) $T_4 = \frac{1}{2}[f(0)+f(1)]\cdot 1 + \frac{1}{2}[f(1)+f(2)]\cdot 1 + \frac{1}{2}[f(2)+f(3)]\cdot 1 + \frac{1}{2}[f(3)+f(4)]\cdot 1$
$= \frac{1}{2}\left[(0-1)^2 + 2(1-1)^2 + 2(2-1)^2 + 2(3-1)^2 + (4-1)^2\right]$

2) $f(x) = x^2+1$ on $[0,4]$

a) $L_4 = 1[f(0) + f(1) + f(2) + f(3)]$
$= 1 + 2 + 5 + 10$

b) $R_4 = 1[f(4) + f(3) + f(2) + f(1)]$
$= 17 + 10 + 5 + 2$

c) $M_4 = 1[f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2})]$
$= \frac{1}{4}+1 + \frac{9}{4}+1 + \frac{25}{4}+1 + \frac{49}{4}+1$

d) $T_4 = \frac{1}{2}(1)[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)]$
$= \frac{1}{2}[1 + 4 + 10 + 20 + 17]$

3) $f(x) = (x+1)^2$ on $[0, 4]$

a) $L_4 = 1[f(0) + f(1) + f(2) + f(3)]$
$= 1 + 4 + 9 + 16$

b) $R_4 = 1[f(4) + f(3) + f(2) + f(1)]$
$= 25 + 16 + 9 + 4$

c) $M_4 = 1[f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2})]$
$= \frac{9}{4} + \frac{25}{4} + \frac{49}{4} + \frac{81}{4}$

d) $T_4 = \frac{1}{2}[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)]$
$= \frac{1}{2}[1 + 2(4) + 2(9) + 2(16) + 25]$

4) $f(x) = \begin{cases} 2x & x \leq 1 \\ 3x^2 - 1 & x > 1 \end{cases}$ $\quad \int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$\int_0^1 2x\,dx + \int_1^2 (3x^2 - 1)\,dx$

$x^2\Big|_0^1 + [x^3 - 1]\Big|_1^2 = 1 + [7] - [0]$
$= \boxed{8}$

5) $\lim_{x \to 4} \frac{x^3 - 4x^2 - x + 4}{x - 4} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x \to 4} \frac{3x^2 - 8x - 1}{1} = \boxed{15}$

6) $x = 4$ by IVT

7) $f(x) = \tan^2(8 - 2x)$
$f(x) = [\tan(8 - 2x)]^2$

$f'(x) = 2[\tan(8 - 2x)] \cdot \sec^2(8 - 2x) \cdot (-2)$

$f'(1) = -4 \tan 6 \sec^2 6$